Step 3:

We want to find the value of $\frac{dr}{dt}$.

Step 4:

From part (b), $r = \sqrt{26y - y^2}$.

Step 5:

$$\frac{dr}{dt} = \frac{1}{2\sqrt{26y - y^2}}(26 - 2y)\frac{dy}{dt}$$
$$= \frac{13 - y}{\sqrt{26y - y^2}}\frac{dy}{dt}$$

Step 6:

$$\frac{dr}{dt} = \frac{13 - 8}{\sqrt{26(8) - 8^2}}\left(\frac{1}{-24\pi}\right)$$
$$= \frac{5}{12}\left(-\frac{1}{24\pi}\right)$$
$$= -\frac{5}{288\pi}$$
$$\approx -0.00553 \text{ m/min}$$

or $-\frac{125}{72\pi} \approx -0.553$ cm/min

19. Step 1:

x = distance from wall to base of ladder
y = height of top of ladder
A = area of triangle formed by the ladder, wall, and ground
θ = angle between the ladder and the ground

Step 2:

At the instant in question, $x = 12$ ft and $\frac{dx}{dt} = 5$ ft/sec.

Step 3:

We want to find $-\frac{dy}{dt}, \frac{dA}{dt},$ and $\frac{d\theta}{dt}$.

Steps 4, 5, and 6:

(a) $x^2 + y^2 = 169$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

To evaluate, note that, at the instant in question,

$$y = \sqrt{169 - x^2} = \sqrt{169 - 12^2} = 5.$$

Then $2(12)(5) + 2(5)\frac{dy}{dt} = 0$

$$\frac{dy}{dt} = -12 \text{ ft/sec} \left(\text{or } -\frac{dy}{dt} = 12 \text{ ft/sec}\right)$$

The top of the ladder is sliding down the wall at the rate of 12 ft/sec. (Note that the *downward* rate of motion is positive.)

(b) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

Using the results from step 2 and from part (a), we have

$$\frac{dA}{dt} = \frac{1}{2}[(12)(-12) + (5)(5)]$$
$$= -\frac{119}{2} \text{ ft}^2/\text{sec}$$

The area of the triangle is changing at the rate of $-59.5 \text{ ft}^2/\text{sec}$.

(c) $\tan\theta = \frac{y}{x}$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

Since $\tan\theta = \frac{5}{12}$, we have $\left(\text{for } 0 \le \theta < \frac{\pi}{2}\right)\cos\theta = \frac{12}{13}$ and so

$$\sec^2\theta\frac{1}{\left(\frac{12}{13}\right)^2} = \frac{169}{144}.$$

Combining this result with the results from step 2 and from part (a), we have

$$\frac{169}{144}\frac{d\theta}{dt} = \frac{(12)(-12) - (5)(5)}{12^2}, \text{ so}$$

$\frac{d\theta}{dt} = -1$ radian/sec. The angle is changing at the rate of -1 radian/sec.

20. Step 1:

h = height (or depth) of the water in the trough
V = volume of water in the trough

Step 2:

At the instant in question, $\frac{dV}{dt} = 2.5 \text{ ft}^3/\text{min}$ and $h = 2$ ft.

Step 3:

We want to find $\frac{dh}{dt}$.

Step 4:

The width of the top surface of the water is $\frac{4}{3}h$, so we have $V = \frac{1}{2}(h)\left(\frac{4}{3}h\right)(15)$, or

$V = 10h^2$
Step 5:
$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$
Step 6:
$$2.5 = 20(2)\frac{dh}{dt}$$
$$\frac{dh}{dt} = 0.0625 = \frac{1}{16} \text{ ft/min}$$
The water level is increasing at the rate of $\frac{1}{16}$ ft/min.

21. Step 1:
l = length of rope
x = horizontal distance from boat to dock
θ = angle between the rope and a vertical line
Step 2:
At the instant in question, $\frac{dl}{dt} = -2$ ft/sec and $l = 10$ ft.
Step 3:
We want to find the values of $-\frac{dx}{dt}$ and $\frac{d\theta}{dt}$.
Steps 4, 5, and 6:

(a) $x = \sqrt{l^2 - 36}$
$$\frac{dx}{dt} = \frac{l}{\sqrt{l^2 - 36}}\frac{dl}{dt}$$
$$\frac{dx}{dt} = \frac{10}{\sqrt{10^2 - 36}}(-2) = -2.5 \text{ ft/sec}$$
The boat is approaching the dock at the rate of 2.5 ft/sec.

(b) $\theta = \cos^{-1}\frac{6}{l}$
$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1 - \left(\frac{6}{l}\right)^2}}\left(-\frac{6}{l^2}\right)\frac{dl}{dt}$$
$$\frac{d\theta}{dt} = -\frac{1}{\sqrt{1 - 0.6^2}}\left(-\frac{6}{10^2}\right)(-2)$$
$$= -\frac{3}{20} \text{ radian/sec}$$
The rate of change of angle θ is $-\frac{3}{20}$ radian/sec.

22. Step 1:
x = distance from origin to bicycle
y = height of balloon (distance from origin to balloon)
s = distance from balloon to bicycle
Step 2:
We know that $\frac{dy}{dt}$ is a constant 1 ft/sec and $\frac{dx}{dt}$ is a constant 17 ft/sec. Three seconds before the instant in question, the values of x and y are $x = 0$ ft and $y = 65$ ft. Therefore, at the instant in question $x = 51$ ft and $y = 68$ ft.
Step 3:
We want to find the value of $\frac{ds}{dt}$ at the instant in question.
Step 4:
$s = \sqrt{x^2 + y^2}$
Step 5:
$$\frac{ds}{dt} = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$
$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$
Step 6:
$$\frac{ds}{dt} = \frac{(51)(17) + (68)(1)}{\sqrt{51^2 + 68^2}} = 11 \text{ ft/sec}$$
The distance between the balloon and the bicycle is increasing at the rate of 11 ft/sec.

23. $$\frac{dy}{dt} = \frac{dy}{dt}\frac{dx}{dt} = -10(1 + x^2)^{-2}(2x)\frac{dx}{dt}$$
$$= -\frac{20x}{(1 + x^2)^2}\frac{dx}{dt}$$
Since $\frac{dx}{dt} = 3$cm/sec, we have
$$\frac{dy}{dt} = -\frac{60x}{(1 + x^2)^2} \text{ cm/sec.}$$

(a) $\frac{dy}{dt} = -\frac{60(-2)}{[1 + (-2)^2]^2} = \frac{120}{5^2} = \frac{24}{5}$ cm/sec

(b) $\frac{dy}{dt} = -\frac{60(0)}{(1 + 0^2)^2} = 0$ cm/sec

(c) $\frac{dy}{dt} = -\frac{60(20)}{(1 + 20^2)^2} \approx -0.00746$ cm/sec

24. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (3x^2 - 4)\frac{dx}{dt}$

Since $\frac{dx}{dt} = -2$ cm/sec, we have

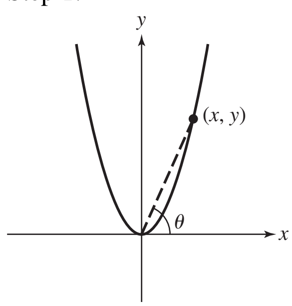
$\frac{dy}{dt} = 8 - 6x^2$ cm/sec.

(a) $\frac{dy}{dt} = 8 - 6(-3)^2 = -46$ cm/sec

(b) $\frac{dy}{dt} = 8 - 6(1)^2 = 2$ cm/sec

(c) $\frac{dy}{dt} = 8 - 6(4)^2 = -88$ cm/sec

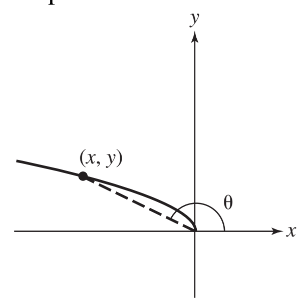
25. Step 1:



x = x-coordinate of particle's location
y = y-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin.
Step 2:
At the instant in question,
$\frac{dx}{dt} = 10$ m/sec and $x = 3$ m.
Step 3:
We want to find $\frac{d\theta}{dt}$.
Step 4:
Since $y = x^2$, we have $\tan\,\theta = \frac{y}{x} = \frac{x^2}{x} = x$

and so, for $x > 0$, $\theta = \tan^{-1} x$.
Step 5:
$\frac{d\theta}{dt} = \frac{1}{1+x^2}\frac{dx}{dt}$
Step 6:
$\frac{d\theta}{dt} = \frac{1}{1+3^2}(10) = 1$ radian/sec
The angle of inclination is increasing at the rate of 1 radian/sec.

26. Step 1:



x = x-coordinate of particle's location
y = y-coordinate of particle's location
θ = angle of inclination of line joining the particle to the origin
Step 2:
At the instant in question, $\frac{dx}{dt} = -8$ m/sec and

$x = -4$ m.
Step 3:
We want to find $\frac{d\theta}{dt}$,
Step 4:
Since $y = \sqrt{-x}$, we have

$\tan\theta = \frac{y}{x} = \frac{\sqrt{-x}}{x} - (-x)^{-1/2}$, and so, for $x < 0$,

$\theta = \pi + \tan^{-1}[-(-x)^{1/2}] = \pi - \tan^{-1}(-x)^{-1/2}$.
Step 5:

$$\frac{d\theta}{dt} = -\frac{1}{1+[(-x)^{-1/2}]^2}\left((-\frac{1}{2}(-x)^{-3/2}(-1)\right)\frac{dx}{dt}$$
$$= -\frac{1}{1-\left(\frac{1}{x}\right)}\frac{1}{2(-x)^{3/2}}\frac{dx}{dt}$$
$$= \frac{1}{2\sqrt{-x}(x-1)}\frac{dx}{dt}$$

Step 6:
$\frac{d\theta}{dt} = \frac{1}{2\sqrt{4}(-4-1)}(-8) = \frac{2}{5}$ radian/sec
The angle of inclination is increasing at the rate of $\frac{2}{5}$ radian/sec.

27. Step 1:
r = radius of balls plus ice
S = surface area of ball plus ice
V = volume of ball plus ice
Step 2:
At the instant in question,

$$\frac{dV}{dt} = -8\,\text{mL/min}$$
$$= -8\,\text{cm}^3/\text{min and } r$$
$$= \frac{1}{2}(20)$$
$$= 10\,\text{cm}.$$

Step 3:

We want to find $-\frac{dS}{dt}$.

Step 4:

We have $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$. These equations can be combined by noting that $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

Step 5:

$$\frac{dS}{dt} = 4\pi\left(\frac{2}{3}\right)\left(\frac{3V}{4\pi}\right)^{-1/3}\left(\frac{3}{4\pi}\right)\frac{dV}{dt}$$
$$= 2\left(\frac{3V}{4\pi}\right)^{-1/3}\frac{dV}{dt}$$

Step 6:

Note that $V = \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3}$.

$$\frac{dS}{dt} = 2\left(\frac{3}{4\pi}\bullet\frac{4000\pi}{3}\right)^{-1/3}(-8)$$
$$= \frac{-16}{\sqrt[3]{1000}}$$
$$= -1.6\,\text{cm}^2/\text{min}$$

Since $\frac{dS}{dt} < 0$, the rate of *decrease* is positive. The surface area is decreasing at the rate of $1.6\,\text{cm}^2/\text{min}$.

28. Step 1:
$x = x$-coordinate of particle
$y = y$-coordinate of particle
$D =$ distance from origin to particle
Step 2:
At the instant in question, $x = 5$ m, $y = 12$ m, $\frac{dx}{dt} = -1$ m/sec, and $\frac{dy}{dt} = -5$ m/sec.
Step 3:
We want to find $\frac{dD}{dt}$.
Step 4:
$D = \sqrt{x^2 + y^2}$
Step 5:

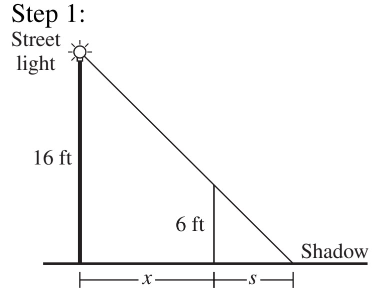
$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$
$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

Step 6:

$$\frac{dD}{dt} = \frac{(5)(-1)+(12)(-5)}{\sqrt{5^2+12^2}} = -5\,\text{m/sec}$$

The particle's distance from the origin is changing at the rate of –5 m/sec.

29. Step 1:



$x =$ distance from streetlight base to man
$s =$ length of shadow
Step 2:
At the instant in question, $\frac{dx}{dt} = -5\,\text{ft/sec}$ and $x = 10$ ft.
Step 3:
We want to find $\frac{ds}{dt}$.
Step 4:
By similar triangles, $\frac{s}{6} = \frac{s+x}{16}$. This is equivalent to $16s = 6s + 6x$, or $s = \frac{3}{5}x$.
Step 5:

$$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt}$$

Step 6:

$$\frac{ds}{dt} = \frac{3}{5}(-5) = -3\,\text{ft/sec}$$

The shadow length is changing at the rate of –3 ft/sec.

30. Step 1:
$s =$ distance ball has fallen
$x =$ distance from bottom of pole to shadow
Step 2:
At the instant in question, $s = 16\left(\frac{1}{2}\right)^2 = 4$ ft

and $\frac{ds}{dt} = 32\left(\frac{1}{2}\right) = 16$ ft/sec.
Step 3:
We want to find $\frac{dx}{dt}$.
Step 4:
By similar triangles, $\frac{x-30}{50-s} = \frac{x}{50}$. This is equivalent to $50x - 1500 = 50x - sx$, or $sx = 1500$. We will use $x = 1500s^{-1}$.
Step 5 :
$\frac{dx}{dt} = -500s^{-2}\frac{ds}{dt}$
Step 6:
$\frac{dx}{dt} = -1500(4)^{-2}(16) = -1500$ ft/sec
The shadow is moving at a velocity of –1500 ft/sec.

31. Step 1:
x = position of car ($x = 0$ when car is right in front of you)
θ = camera angle. (We assume θ is negative until the car passess in front of you, and then positive.)
Step 2:
At the first instant in question, $x = 0$ ft and
$\frac{dx}{dt} = 264$ ft/sec.
A half second later, $x = \frac{1}{2}(264) = 132$ ft and
$\frac{dx}{dt} = 264$ ft/sec.
Step 3:
We want to find $\frac{d\theta}{dt}$ at each of the two instants.
Step 4:
$\theta = \tan^{-1}\left(\frac{x}{132}\right)$
Step 5:
$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{132}\right)^2}\cdot\frac{1}{132}\frac{dx}{dt}$
Step 6:
When $x = 0$:
$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{0}{132}\right)^2}\left(\frac{1}{132}\right)(264) = 2$ radians/sec
When $x = 132$:
$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{132}{132}\right)^2}\left(\frac{1}{132}\right)(264) = 1$ radians/sec

32. Step 1:
p = x-coordinate of plane's position
x = x-coordinate of car's position
s = distance from plane to car (line-of-sight)
Step 2:
At the instant in question, $p = 0$,
$\frac{dp}{dt} = 120$ mph, $s = 5$ mi, and $\frac{ds}{dt} = -160$ mph.
Step 3:
We want to find $-\frac{dx}{dt}$.
Step 4:
$(x-p)^2 + 3^2 = s^2$
Step 5:
$2(x-p)\left(\frac{dx}{dt} - \frac{dp}{dt}\right) = 2s\frac{ds}{dt}$
Step 6:
Note that, at the instant in question,
$x = \sqrt{5^2 - 3^2} = 4$ mi.

$$2(4-0)\left(\frac{dx}{dt} - 120\right) = 2(5)(-160)$$
$$8\left(\frac{dx}{dt} - 120\right) = -1600$$
$$\frac{dx}{dt} - 120 = -200$$
$$\frac{dx}{dt} = -80 \text{ mph}$$

The car's speed is 80 mph.

33. Step 1:
s = shadow length
θ = sun's angle of elevation
Step 2:
At the instant in question, $s = 60$ ft and
$\frac{d\theta}{dt} = 0.27°/\min = 0.0015\pi$ radian/min.
Step 3:
We want to find $-\frac{ds}{dt}$.
Step 4:
$\tan\theta = \frac{80}{s}$ or $s = 80\cot\theta$
Step 5:
$\frac{ds}{dt} = -80\csc^2\theta\frac{d\theta}{dt}$

Step 6:
Note that, at the moment in question, since

$\tan\theta = \frac{80}{60}$ and $0 < \theta < \frac{\pi}{2}$, we have

$\sin\theta = \frac{4}{5}$ and so $\csc\theta = \frac{5}{4}$.

$$\begin{aligned}\frac{ds}{dt} &= -80\left(\frac{5}{4}\right)^2 (0.0015\pi) \\ &= -0.1875\pi \frac{\text{ft}}{\text{min}} \cdot \frac{12\text{ in}}{1\text{ ft}} \\ &= -2.25\pi \text{ in./min} \\ &\approx -7.1 \text{ in./min}\end{aligned}$$

Since $\frac{ds}{dt} < 0$, the rate at which the shadow length is *decreasing* is positive. The shadow length is decreasing at the rate of approximately 7.1 in./min.

34. Step 1:
$a =$ distance from origin to A
$b =$ distance from origin to B
$\theta =$ angle shown in problem statement
Step 2:
At the instant in question,
$\frac{da}{dt} = -2\text{m/sec}, \frac{db}{dt} = 1 \text{ m/sec},$
$a = 10\text{m}$, and $b = 20\text{m}$.
Step 3:
We want to find $\frac{d\theta}{dt}$.
Step 4:
$\tan\theta = \frac{a}{b}$ or $\theta = \tan^{-1}\left(\frac{a}{b}\right)$
Step 5:

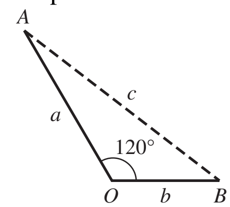
$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{a}{b}\right)^2} \frac{b\frac{da}{dt} - a\frac{db}{dt}}{b^2} = \frac{b\frac{da}{dt} - a\frac{db}{dt}}{a^2 + b^2}$$

Step 6:

$$\begin{aligned}\frac{d\theta}{dt} &= \frac{(20)(-2)-(10)(1)}{10^2 + 20^2} \\ &= -0.1 \text{radian / sec} \\ &\approx -5.73 \text{ degrees/sec}\end{aligned}$$

To the nearest degree, the angle is changing at the rate of –6 degrees per second.

35. Step 1:



$a =$ distance from O to A
$b =$ distance from O to B
$c =$ distance from A to B
Step 2:
At the instant in question, $a = 5$ nautical miles, $b = 3$ nautical miles,
$\frac{da}{dt} = 14$ knots, and $\frac{db}{dt} = 21$ knots.
Step 3:
We want to find $\frac{dc}{dt}$,
Step 4:
Law of Cosines :
$c^2 = a^2 + b^2 - 2ab\cos 120°$
$c^2 = a^2 + b^2 + ab$
Step 5:

$$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} + a\frac{db}{dt} + b\frac{da}{dt}$$

Step 6:
Note that, at the instant in question,

$$\begin{aligned}c &= \sqrt{a^2 + b^2 + ab} \\ &= \sqrt{(5)^2 + (3)^2 + (5)(3)} \\ &= \sqrt{49} \\ &= 7\end{aligned}$$

$$\begin{aligned}2(7)\frac{dc}{dt} &= 2(5)(14) + 2(3)(21) + (5)(21) + (3)(14) \\ 14\frac{dc}{dt} &= 413 \\ \frac{dc}{dt} &= 29.5 \text{ knots}\end{aligned}$$

The ships are moving apart at a rate of 29.5 knots.

36. True. Since $\frac{dC}{dt} = 2\pi\frac{dr}{dt}$, a constant $\frac{dr}{dt}$ results in a constant $\frac{dC}{dt}$.

37. False. Since $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$, the value of $\frac{dA}{dt}$ depends on r.

38. A; $V = s^3$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$
$$24 = 3s^2(2)$$
$$s = 2\,\text{in}$$

39. E; $sA = 6s^2$

$$\frac{dsA}{dt} = 12s\frac{ds}{dt}$$
$$12 = 12s\frac{ds}{dt}$$
$$\frac{ds}{dt} = \frac{1}{s}$$
$$V = s^3$$
$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt} = 3s^2 \frac{1}{s}$$
$$24 = 3s$$
$$s = 8 \text{ in}$$

40. C; $x^2 + y^2 = 1$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$x\frac{dx}{dt} = -y\frac{dy}{dt}$$
$$\frac{x}{-y}\frac{dx}{dt} = \frac{dy}{dt}$$
$$\left(\frac{0.6}{-0.8}\right)3 = \frac{dy}{dt}$$
$$\frac{dy}{dt} = -2.25.$$

41. B; $v = \pi r^2 l$

$$\frac{dv}{dt} = 2\pi rl\frac{dr}{dt} + \pi r^2 \frac{dl}{dt}$$
$$0 = 2\pi(1)(100)\frac{dr}{dt} + \pi(1)^2 2$$
$$\frac{dr}{dt} = \frac{-2\pi}{200\pi}$$
$$\frac{dr}{dt} = -.01 \text{ cm/s}$$

42. **(a)** Note that the level of the coffee in the cone is not needed until part (b).

Step 1:
V_1 = volume of coffee in pot
y = depth of coffee in pot

Step 2:
$$\frac{dV_1}{dt} = 10 \text{ in}^3/\text{min}$$

Step 3:
We want to find the value of $\frac{dy}{dt}$.

Step 4:
$V_1 = 9\pi y$

Step 5:
$$\frac{dV_1}{dt} = 9\pi\frac{dy}{dt}$$

Step 6:
$$10 = 9\pi\frac{dy}{dt}$$
$$\frac{dy}{dt} = \frac{10}{9\pi} \approx 0.354 \text{ in./min}$$

The level in the pot is increasing at the rate of approximately 0.354 in./min.

(b) Step 1:
V_2 = volume of coffee in filter
r = radius of surface of coffee in filter
h = depth of coffee in filter

Step 2:
At the instant in question,
$$\frac{dV_2}{dt} = -10 \text{ in}^3/\text{min} \text{ and } h = 5 \text{ in.}$$

Step 3:
We want to find $-\frac{dh}{dt}$.

Step 4:
Note that $\frac{r}{h} = \frac{3}{6}$, so $r = \frac{h}{2}$.

Then $V_2 = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$.

Step 5:
$$\frac{dV_2}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$$

Step 6:
$$-10 = \frac{\pi(5)^2}{4}\frac{dh}{dt}$$
$$\frac{dh}{dt} = -\frac{8}{5\pi} \text{in./min}$$

Note that $\frac{dh}{dt} < 0$, so the rate at which the level is *falling* is positive. The level in the come is falling at the rate of $\frac{8}{5\pi} \approx 0.509$ in./min.

43. **(a)** $\frac{dc}{dt} = \frac{d}{dt}(x^3 - 6x^2 + 15x)$

$= (3x^2 - 12x + 15)\frac{dx}{dt}$

$= [3(2)^2 - 12(2) + 15](0.1)$

$= 0.3$

$\frac{dr}{dt} = \frac{d}{dt}(9x) = 9\frac{dx}{dt} = 9(0.1) = 0.9$

$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 0.9 - 0.3 = 0.6$

(b) $\frac{dc}{dt} = \frac{d}{dt}\left(x^3 - 6x^2 + \frac{45}{x}\right)$

$= \left(3x^2 - 12x - \frac{45}{x^2}\right)\frac{dx}{dt}$

$= \left[3(1.5)^2 - 12(1.5) - \frac{45}{1.5^2}\right](0.05)$

$= -1.5625$

$\frac{dr}{dt} = \frac{d}{dt}(70x) = 70\frac{dx}{dt} = 70(0.05) = 3.5$

$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 3.5 - (-1.5625) = 5.0625$

44. Step 1:
Q = rate of CO_2 exhalation (mL/min)
D = difference between CO_2 concentration in blood pumped to the lungs and CO_2 concentration in blood returning from the lungs (mL/L)
y = cardiac output
Step 2:
At the instant in question, $Q = 233$ mL/min, $D = 41$ mL/L, $\frac{dD}{dt} = -2$ (mL/L)/min, and $\frac{dQ}{dt} = 0$ mL/min^2.
Step 3:
We want to find the value of $\frac{dy}{dt}$.
Step 4:
$y = \frac{Q}{D}$
Step 5:
$\frac{dy}{dt} = \frac{D\frac{dQ}{dt} - Q\frac{dD}{dt}}{D^2}$
Step 6:
$\frac{dy}{dt} = \frac{(41)(0) - (233)(-2)}{(41)^2}$

$= \frac{466}{1681}$

≈ 0.277 L/min^2

The cardiac output is increasing at the rate of approximately 0.277 L/min^2.

45. **(a)** The point being plotted would correspond to a point on the edge of the wheel as the wheel turns.

(b) One possible answer is $\theta = 16\pi t$, where t is in seconds. (An arbitrary constant may be added to this expression, and we have assumed counterclockwise motion.)

(c) In general, assuming counterclockwise motion:

$\frac{dx}{dt} = -2\sin\theta\frac{d\theta}{dt}$

$= -2(\sin\theta)(16\pi)$

$= -32\pi\sin\theta$

$\frac{dy}{dt} = 2\cos\theta\frac{d\theta}{dt}$

$= 2(\cos\theta)(16\pi)$

$= 32\pi\cos\theta$

At $\theta = \frac{\pi}{4}$:

$\frac{dx}{dt} = -32\pi\sin\frac{\pi}{4}$

$= -16\pi(\sqrt{2})$

≈ -71.086 ft/sec

$\frac{dy}{dt} = 32\pi\cos\frac{\pi}{4}$

$= 16\pi(\sqrt{2})$

≈ 71.086 ft/sec

At $\theta = \frac{\pi}{2}$:

$\frac{dx}{dt} = -32\pi\sin\frac{\pi}{2}$

$= -32\pi$

≈ -100.531 ft/sec

$\frac{dy}{dt} = 32\pi\cos\frac{\pi}{2} = 0$ ft/sec

At $\theta = \pi$:

$\frac{dx}{dt} = -32\pi\sin\pi = 0$ ft/sec

$\frac{dy}{dt} = 32\pi\cos\pi = -32\pi \approx -100.531$ ft/sec

46. **(a)** One possible answer: $y = 30\cos\theta$, $y = 40 + 30\sin\theta$

(b) Since the ferris wheel makes one revolution every 10 sec, we may let $\theta = 0.2\pi t$ and we may write $x = 30\cos 0.2\pi t,\ y = 40 + 30\sin 0.2\pi t$. (This assumes that the ferris wheel revolves counterclockwise.)
In general:

$$\frac{dx}{dt} = -30(\sin 0.2\pi t)(0.2\pi) = -6\pi \sin 0.2\pi t$$

$$\frac{dy}{dt} = 30(\cos 0.2\pi t)(0.2\pi) = 6\pi\cos 0.2\pi t$$

At $t = 5$:

$$\frac{dx}{dt} = -6\pi\sin\pi = 0 \text{ ft/sec}$$

$$\frac{dy}{dt} = 6\pi\cos\pi = 6\pi(-1) \approx -18.850 \text{ ft/sec}$$

At $t = 8$:

$$\frac{dx}{dt} = -6\pi\sin 1.6\pi \approx 17.927 \text{ ft/sec}$$

$$\frac{dy}{dt} = 6\pi\cos 1.6\pi \approx 5.825 \text{ ft/sec}$$

47. **(a)**

$$\frac{dy}{dt} = \frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt} = u(0.05v) + v(0.04u) = 0.09uv = 0.09y$$

Since $\frac{dy}{dt} = 0.09y$, the rate of growth of total production is 9% per year.

(b)

$$\frac{dy}{dt} = \frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt} = u(0.03v) + v(-0.02u) = 0.01uv = 0.01y$$

The total production is increasing at the rate of 1% per year.

Quick Quiz Sections 5.4–5.6

1. B; $x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$

$$f(x) = x^3 + 2x - 1$$

$$f'(x) = 3x^2 + 2$$

$$x_2 = 1 - \frac{(1)^3 + 2(1) - 1}{3(1)^2 + 2} = \frac{3}{5}$$

$$x_3 = \frac{3}{5} - \frac{\left(\frac{3}{5}\right)^3 + 2\left(\frac{3}{5}\right) - 1}{3\left(\frac{3}{5}\right)^2 + 2} = 0.465$$

2. B; $z^2 = x^2 + y^2$

$$z = \sqrt{4^2 + 3^2} = 5$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$5 = 4\left(3\frac{dy}{dt}\right) + 3\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{1}{3}$$

$$\frac{dx}{dt} = 3\frac{dy}{dt} = 3\left(\frac{1}{3}\right) = 1$$

3. A; $x(t) = 70$

$$y(t) = 60t$$

$$z(t) = ((60t)^2 + 70^2)^{1/2}$$

$$\frac{dz}{dt} = \frac{1}{2}(3600t^2 + 4900)^{-1/2}(7200t)$$

$$\frac{dz}{dt} = \frac{7200(4)}{2(3600(4)^2 + 4900)^{1/2}}$$

$$\frac{dz}{dt} = 57.6$$

4. **(a)** $f(x) = \sqrt{x}$

$$x = 25$$

$$f'(25) = \frac{1}{2}(25)^{-1/2} = \frac{1}{10}$$

$$\sqrt{26} = 5 + \frac{1}{10}(26 - 25) = 5.1$$

(b) $x_{n+1} = x_n - \dfrac{f(x)}{f'(x)},\ f(x) = x^2 - 26 = 0$

$$x_2 = 5 - \frac{(5)^2 - 26}{2(5)} = 5.1$$

(c) $f(x) = \sqrt[3]{x}$

$x = 3$

$f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$

$\sqrt{26} = 3 + \frac{1}{27}(26 - 27)$

$\sqrt{26} = 2.963$

Chapter 5 Review Exercises (pp. 260–264)

1. $y = x\sqrt{2-x}$

$$y' = x\left(\frac{1}{2\sqrt{2-x}}\right)(-1) + (\sqrt{2-x})(1)$$
$$= \frac{-x + 2(2-x)}{2\sqrt{2-x}}$$
$$= \frac{4-3x}{2\sqrt{2-x}}$$

The first derivative has a zero at $\frac{4}{3}$.

Critical point value: $x = \frac{4}{3}$ $\quad y = \frac{4\sqrt{6}}{9}$

Endpoint values: $x = -2$ $\quad y = -4$

$x = 2$ $\quad y = 0$

The global maximum value is $\frac{4\sqrt{6}}{9}$ at $x = \frac{4}{3}$, and the global minimum value is –4 at $x = -2$.

2. Since y is a cubic function with a positive leading coefficient, we have $\lim_{x\to-\infty} y = -\infty$ and $\lim_{x\to\infty} y = \infty$. There are no global extrema.

3. $y' = (x^2)(e^{1/x^2})(-2x^{-3}) + (e^{1/x^2})(2x)$

$$= 2e^{1/x^2}\left(-\frac{1}{x} + x\right)$$
$$= \frac{2e^{1/x^2}(x-1)(x+1)}{x}$$

Intervals	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
Sign of y'	–	+	–	+
Behavior of y	Decreasing	Increasing	Decreasing	Increasing

$$y'' = \frac{d}{dx}[2e^{1/x^2}(-x^{-1}+x)]$$
$$= (2e^{1/x^2})(x^{-2}+1) + (-x^{-1}+x)(2e^{1/x^2})(-2x^{-3})$$
$$= (2e^{1/x^2})(x^{-2}+1+2x^{-4}-2x^{-2})$$
$$= \frac{2e^{1/x^2}(x^4-x^2+2)}{x^4}$$
$$= \frac{2e^{1/x^2}[(x^2-0.5)^2+1.75]}{x^4}$$

The second derivative is always positive (where defined), so the function is concave up for all $x \neq 0$.

(a) $[-1,\ 0)$ and $[1,\ \infty)$

(b) $(-\infty, -1]$ and $(0,\ 1]$

(c) $(-\infty,\ 0)$ and $(0,\ \infty)$

(d) None

(e) Local (and absolute) minima at $(1,\ e)$ and $(-1,\ e)$

(f) None

4. Note that the domain of the function is $[-2,\ 2]$.

$$y' = x\left(\frac{1}{2\sqrt{4-x^2}}\right)(-2x) + \left(\sqrt{4-x^2}\right)(1)$$
$$= \frac{-x^2+(4-x^2)}{\sqrt{4-x^2}}$$
$$= \frac{4-2x^2}{\sqrt{4-x^2}}$$

Intervals	$-2 < x < -\sqrt{2}$	$-\sqrt{2} < x < \sqrt{2}$	$\sqrt{2} < x < 2$
Sign of y'	$-$	$+$	$-$
Behavior of y	Decreasing	Increasing	Decreasing

$$y'' = \frac{(\sqrt{4-x^2})(-4x) - (4-2x^2)\left(\frac{1}{2\sqrt{4-x^2}}\right)(-2x)}{4-x^2}$$
$$= \frac{2x(x^2-6)}{(4-x^2)^{3/2}}$$

Note that the values $x = \pm\sqrt{6}$ are not zeros of y'' because they fall outside of the domain.

Intervals	$-2 < x < 0$	$0 < x < 2$
Sign of y''	$+$	$-$
Behavior of y	Concave up	Concave down

(a) $\left[-\sqrt{2}, \sqrt{2}\right]$

(b) $\left[-2, -\sqrt{2}\right]$ and $\left[\sqrt{2}, 2\right]$

(c) $(-2, 0)$

(d) $(0, 2)$

(e) Local maxima: $(-2, 0)$, $(\sqrt{2}, 2)$
Local minima: $(2, 0)$, $(-\sqrt{2}, -2)$
Note that the extrema at $x = \pm\sqrt{2}$ are also absolute extrema.

(f) $(0, 0)$

5. $y' = 1 - 2x - 4x^3$
Using grapher techniques, the zero of y' is $x \approx 0.385$.

Intervals	$x < 0.385$	$0.385 < x$
Sign of y'	+	–
Behavior of y	Increasing	Decreasing

$y'' = -2 - 12x^2 = -2(1 + 6x^2)$
The second derivative is always negative so the function is concave down for all x.

(a) Approximately $(-\infty, 0.385]$

(b) Approximately $[0.385, \infty)$

(c) None

(d) $(-\infty, \infty)$

(e) Local (and absolute) maximum at $\approx (0.385, 1.215)$

(f) None

6. $y' = e^{x-1} - 1$

Intervals	$x < 1$	$1 < x$
Sign of y'	–	+
Behavior of y	Decreasing	Increasing

$y'' = e^{x-1}$
The second derivative is always positive, so the function is concave up for all x.

(a) $[1, \infty)$

(b) $(-\infty, 1]$

(c) $(-\infty, \infty)$

(d) None

(e) Local (and absolute) minimum at $(1, 0)$

(f) None

7. Note that the domain is $(-1, 1)$.
$y = (1 - x^2)^{-1/4}$
$y' = -\frac{1}{4}(1 - x^2)^{-5/4}(-2x) = \frac{x}{2(1 - x^2)^{5/4}}$

Intervals	$-1 < x < 0$	$0 < x < 1$
Sign of y'	–	+
Behavior of y	Decreasing	Increasing

$$y'' = \frac{2(1-x^2)^{5/4}(1) - (x)(2)\left(\frac{5}{4}\right)(1-x^2)^{1/4}(-2x)}{4(1-x^2)^{5/2}}$$
$$= \frac{(1-x^2)^{1/4}[2 - 2x^2 + 5x^2]}{4(1-x^2)^{5/2}}$$
$$= \frac{3x^2 + 2}{4(1-x^2)^{9/4}}$$

The second derivative is always positive, so the function is concave up on its domain $(-1, 1)$.

(a) $[0, 1)$

(b) $(-1, 0]$

(c) $(-1, 1)$

(d) None

(e) Local minimum at $(0, 1)$

(f) None

8. $y' = \dfrac{(x^3-1)(1)-(x)(3x^2)}{(x^3-1)^2} = \dfrac{-(2x^3+1)}{(x^3-1)^2}$

Intervals	$x < -2^{-1/3}$	$-2^{-1/3} < x < 1$	$1 < x$
Sign of y'	+	−	−
Behavior of y	Increasing	Decreasing	Decreasing

$$y'' = -\frac{(x^3-1)^2(6x^2)-(2x^3+1)(2)(x^3-1)(3x^2)}{(x^3-1)^4}$$
$$= -\frac{(x^3-1)(6x^2)-(2x^3+1)(6x^2)}{(x^3-1)^3}$$
$$= \frac{6x^2(x^3+2)}{(x^3-1)^3}$$

Intervals	$x < -2^{1/3}$	$-2^{1/3} < x < 0$	$0 < x < 1$	$1 < x$
Sign of y''	+	−	−	+
Behavior of y	Concave up	Concave down	Concave down	Concave up

(a) $(-\infty, -2^{+1/3}] \approx (-\infty, -0.794]$

(b) $[-2^{+1/3}, 1) \approx [-0.794, 1)$ and $(1, \infty)$

(c) $(-\infty, -2^{+1/3}) \approx (-\infty, -1.260)$ and $(1, \infty)$

(d) $(-2^{+1/3}, 1) \approx (-1.260, 1)$

(e) Local minimum at
$\left(-2^{-1/3}, \dfrac{2}{3}\cdot 2^{-1/3}\right) \approx (-0.794, 0.529)$

(f) $\left(-2^{1/3}, \dfrac{1}{3}\cdot 2^{1/3}\right) \approx (-1.260, 0.420)$

9. Note that the domain is $[-1, 1]$.

$y' = -\dfrac{1}{\sqrt{1-x^2}}$

Since y' is negative on $(-1, 1)$ and y is continuous, y is decreasing on its domain $[-1, 1]$.

$$y'' = \frac{d}{dx}[-(1-x^2)^{-1/2}]$$
$$= \frac{1}{2}(1-x^2)^{-3/2}(-2x)$$
$$= -\frac{x}{(1-x^2)^{3/2}}$$

Intervals	$-1 < x < 0$	$0 < x < 1$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

(a) None

(b) $[-1, 1]$

(c) $(-1, 0)$

(d) $(0, 1)$

(e) Local (and absolute) maximum at $(-1, \pi)$;
local (and absolute) minimum at $(1, 0)$

(f) $\left(0, \dfrac{\pi}{2}\right)$

10. Note that the denominator of y is always positive because it is equivalent to $(x+1)^2 + 2$.

$$y' = \frac{(x^2+2x+3)(1)-(x)(2x+2)}{(x^2+2x+3)^2}$$
$$= \frac{-x^2+3}{(x^2+2x+3)^2}$$

Intervals	$x < -\sqrt{3}$	$-\sqrt{3} < x < \sqrt{3}$	$\sqrt{3} < x$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$$y'' = \frac{(x^2+2x+3)^2(-2x)-(-x^2+3)(2)(x^2+2x+3)(2x+2)}{(x^2+2x+3)^4}$$
$$= \frac{(x^2+2x+3)(-2x)-2(2x+2)(-x^2+3)}{(x^2+2x+3)^3}$$
$$= \frac{2x^3-18x-12}{(x^2+2x+3)^3}$$

Using graphing techniques, the zeros of $2x^3-18x-12$ (and hence of y'') are at $x \approx -2.584$, $x \approx -0.706$, and $x \approx 3.290$.

Intervals	$(-\infty, -2.584)$	$(-2.584, -0.706)$	$(-0.706, 3.290)$	$(3.290, \infty)$
Sign of y''	−	+	−	+
Behavior of y	Concave down	Concave up	Concave down	Concave up

(a) $\left[-\sqrt{3}, \sqrt{3}\right]$

(b) $\left(-\infty, -\sqrt{3}\right]$ and $\left[\sqrt{3}, \infty\right)$

(c) Approximately $(-2.584, -0.706)$ and $(3.290, \infty)$

(d) Approximately $(-\infty, -2.584)$ and $(-0.706, 3.290)$

(e) Local maximum at $\left(\sqrt{3}, \dfrac{\sqrt{3}-1}{4}\right) \approx (1.732, 0.183)$;

local minimum at $\left(-\sqrt{3}, \dfrac{-\sqrt{3}-1}{4}\right) \approx (-1.732, -0.683)$

(f) $\approx(-2.584, -0.573)$, $(-0.706, -0.338)$, and $(3.290, 0.161)$

11. For $x > 0$, $y' = \dfrac{d}{dx} \ln x = \dfrac{1}{x}$

For $x < 0$: $y' = \dfrac{d}{dx}\ln(-x) = \dfrac{1}{-x}(-1) = \dfrac{1}{x}$

Thus $y' = \dfrac{1}{x}$ for all x in the domain.

Intervals	$(-2, 0)$	$(0, 2)$
Sign of y'	$-$	$+$
Behavior of y	Decreasing	Increasing

$y'' = -x^{-2}$.

The second derivative is always negative, so the function is concave down on each open interval of its domain.

(a) $(0, 2]$

(b) $[-2, 0)$

(c) None

(d) $(-2, 0)$ and $(0, 2)$

(e) Local (and absolute) maxima at $(-2, \ln 2)$ and $(2, \ln 2)$

(f) None

12. $y' = 3\cos 3x - 4\sin 4x$

Using graphing techniques, the zeros of y' in the domain $0 \le x \le 2\pi$ are $x \approx 0.176$, $x \approx 0.994$,

$x = \dfrac{\pi}{2} \approx 1.57$, $x \approx 2.148$, and $x \approx 2.965$, $x \approx 3.834$, $x = \dfrac{3\pi}{2}$, $x \approx 5.591$

Intervals	$0 < x < 0.176$	$0.176 < x < 0.994$	$0.994 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < 2.148$	$2.148 < x < 2.965$
Sign of y'	$+$	$-$	$+$	$-$	$+$
Behavior of y	Increasing	Decreasing	Increasing	Decreasing	Increasing

Intervals	$2.965 < x < 3.834$	$3.834 < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 5.591$	$5.591 < x < 2\pi$
Sign of y'	−	+	−	+
Behavior of y	Decreasing	Increasing	Decreasing	Increasing

$y'' = -9 \sin 3x - 16 \cos 4x$

Using graphing techniques, the zeros of y'' in the domain $0 \le x \le 2\pi$ are $x \approx 0.542$, $x \approx 1.266$, $x \approx 1.876$, $x \approx 2.600$, $x \approx 3.425$, $x \approx 4.281$, $x \approx 5.144$ and $x \approx 6.000$.

Intervals	$0 < x < 0.542$	$0.542 < x < 1.266$	$1.266 < x < 1.876$	$1.876 < x < 2.600$	$2.600 < x < 3.425$
Sign of y''	−	+	−	+	−
Behavior of y	Concave down	Concave up	Concave down	Concave up	Concave down

Intervals	$3.425 < x < 4.281$	$4.281 < x < 5.144$	$5.144 < x < 6.000$	$6.00 < x < 2\pi$
Sign of y''	+	−	+	−
Behavior of y	Concave up	Concave down	Concave up	Concave down

(a) Approximately $[0, 0.176]$, $\left[0.994, \frac{\pi}{2}\right]$, $[2.148, 2.965]$, $\left[3.834, \frac{3\pi}{2}\right]$, and $[5.591, 2\pi]$

(b) Approximately $[0.176, 0.994]$, $\left[\frac{\pi}{2}, 2.148\right]$, $[2.965, 3.834]$, and $\left[\frac{3\pi}{2}, 5.591\right]$

(c) Approximately (0.542, 1.266), (1.876, 2.600), (3,425, 4.281), and (5.144, 6.000)

(d) Approximately (0, 0.542), (1.266, 1.876), (2.600, 3.425), (4.281, 5.144), and $(6.000, 2\pi)$

(e) Local maxima at $\approx (0.176, 1.266)$, $\left(\frac{\pi}{2}, 0\right)$ and $(2.965, 1.266)$, $\left(\frac{3\pi}{2}, 2\right)$, and $(2\pi, 1)$;

local minima at $\approx (0, 1)$, $(0.994, -0.513)$, $(2.148, -0.513)$, $(3.834, -1.806)$, and $(5.591, -1.806)$

Note that the local extrema at $x \approx 3.834$, $x = \frac{3\pi}{2}$, and $x \approx 5.591$ are also absolute extrema.

(f) $\approx (0.542, 0.437)$, $(1.266, -0.267)$, $(1.876, -0.267)$, $(2.600, 0.437)$, $(3.425, -0.329)$, $(4.281, 0.120)$, $(5.144, 0.120)$, and $(6.000, -0.329)$

13. $y' = \begin{cases} -e^{-x}, & x < 0 \\ 4 - 3x^2, & x > 0 \end{cases}$

Intervals	$x < 0$	$0 < x < \frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} < x$
Sign of y' y'	–	+	–
Behavior of y	Decreasing	Increasing	Decreasing

$y'' = \begin{cases} e^{-x}, & x < 0 \\ -6x, & x > 0 \end{cases}$

Intervals	$x < 0$	$0 < x$
Sign of y''	+	–
Behavior of y	Concave up	Concave down

(a) $\left(0, \frac{2}{\sqrt{3}}\right]$

(b) $(-\infty,\ 0]$ and $\left[\frac{2}{\sqrt{3}}, \infty\right)$

(c) $(-\infty, 0)$

(d) $(0,\ \infty)$

(e) Local maximum at $\left(\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right) \approx (1.155,\ 3.079)$

(f) None. Note that there is no point of inflection at $x = 0$ because the derivative is undefined and no tangent line exists at this point.

14. $y' = -5x^4 + 7x^2 + 10x + 4$

Using graphing techniques, the zeros of y' are $x \approx -0.578$ and $x \approx -1.692$.

Intervals	$x < -0.578$	$-0.578 < x < 1.692$	$1.692 < x$
Sign of y'	–	+	–
Behavior of y	Decreasing	Increasing	Decreasing

$y'' = -20x^3 + 14x + 10$

Using graphing techniques, the zero of y'' is $x \approx 1.079$.

Intervals	$x < 1.079$	$1.079 < x$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

(a) Approximately $[-0.578, 1.692]$

(b) Approximately $(-\infty, -0.578]$ and $[1.692, \infty)$

(c) Approximately $(-\infty, 1.079)$

(d) Approximately $(1.079, \infty)$

(e) Local maximum at $\approx (1.692, 20.517)$; local minimum at $\approx (-0.578, 0.972)$

(f) $\approx (1.079, 13.601)$

15. $y = 2x^{4/5} - x^{9/5}$

$$y' = \frac{8}{5}x^{-1/5} - \frac{9}{5}x^{4/5} = \frac{8-9x}{5\sqrt[5]{x}}$$

Intervals	$x < 0$	$0 < x < \frac{8}{9}$	$\frac{8}{9} < x$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$$y'' = -\frac{8}{25}x^{-6/5} - \frac{36}{25}x^{-1/5} = \frac{-4(2+9x)}{25x^{6/5}}$$

Intervals	$x < -\frac{2}{9}$	$-\frac{2}{9} < x < 0$	$0 < x$
Sign of y''	+	−	−
Behavior of y	Concave up	Concave down	Concave down

(a) $\left[0, \frac{8}{9}\right]$

(b) $(-\infty, 0]$ and $\left[\frac{8}{9}, \infty\right)$

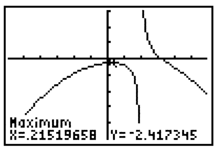
(c) $\left(-\infty, -\frac{2}{9}\right)$

(d) $\left(-\frac{2}{9}, 0\right)$ and $(0, \infty)$

(e) Local maximum at $\left(\frac{8}{9}, \frac{10}{9}\cdot\left(\frac{8}{9}\right)^{4/5}\right) \approx (0.889,\ 1.011)$; local minimum at $(0, 0)$

(f) $\left(-\frac{2}{9}, \frac{20}{9}\cdot\left(-\frac{2}{9}\right)^{4/5}\right) \approx \left(-\frac{2}{9},\ 0.667\right)$

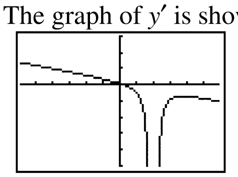
16. We use a combination of analytic and grapher techniques to solve this problem. Depending on the viewing windows chosen, graphs obtained using NDER may exhibit strange behavior near $x = 2$ because, for example, $\text{NDER}(y, 2) \approx 5{,}000{,}000$ while y' is actually undefined at $x = 2$. The graph of $y = \frac{5-4x+4x^2-x^3}{x-2}$ is shown below.



[–5.875, 5.875] by [–50, 30]

$$y' = \frac{(x-2)(-4+8x-3x^2)-(5-4x+4x^2-x^3)(1)}{(x-2)^2}$$
$$= \frac{-2x^3+10x^2-16x+3}{(x-2)^2}$$

The graph of y' is shown below.



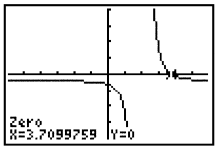
[–5.875, 5.875] by [–50, 30]

The zero of y' is $x \approx 0.215$.

Intervals	$x < 0.215$	$0.215 < x < 2$	$2 < x$
Sign of y'	+	–	–
Behavior of y	Increasing	Decreasing	Decreasing

$$y'' = \frac{(x-2)^2(-6x^2+20x-16)-(-2x^3+10x^2-16x+3)(2)(x-2)}{(x-2)^4}$$
$$= \frac{(x-2)(-6x^2+20x-16)-2(-2x^3+10x^2-16x+3)}{(x-2)^3}$$
$$= \frac{-2(x^3-6x^2+12x-13)}{(x-2)^3}$$

The graph of y'' is shown on the next page.



[–5.875, 5.875] by [–20, 20]

The zero of $x^3 - 6x^2 + 12x - 13$ (and hence of y'') is $x \approx 3.710$.

Intervals	$x < 2$	$2 < x < 3.710$	$3.710 < x$
Sign of y''	–	+	–
Behavior of y	Concave down	Concave up	Concave down

(a) Approximately $(-\infty, 0.215]$

(b) Approximately $[0.215, 2)$ and $(2, \infty)$

(c) Approximately $(2, 3.710)$

(d) $(-\infty, 2)$ and approximately $(3.710, \infty)$

(e) Local maximum at $\approx (0.215, -2.417)$

(f) $\approx(3.710, -3.420)$

17. $y' = 6(x+1)(x-2)^2$

Intervals	$x < -1$	$-1 < x < 2$	$2 < x$
Sign of y'	–	+	+
Behavior of y	Decreasing	Increasing	Increasing

$$\begin{aligned} y'' &= 6(x+1)(2)(x-2) + 6(x-2)^2(1) \\ &= 6(x-2)[(2x+2)+(x-2)] \\ &= 18x(x-2) \end{aligned}$$

Intervals	$x < 0$	$0 < x < 2$	$2 < x$
Sign of y''	+	–	+
Behavior of y	Concave up	Concave down	Concave up

(a) There are no local maxima.

(b) There is a local (and absolute) minimum at $x = -1$.

(c) There are points of inflection at $x = 0$ and at $x = 2$.

18. $y' = 6(x+1)(x-2)$

Intervals	$x < -1$	$-1 < x < 2$	$2 < x$
Sign of y'	+	−	+
Behavior of y	Increasing	Decreasing	Increasing

$$y'' = \frac{d}{dx}6(x^2 - x - 2) = 6(2x-1)$$

Intervals	$x < \frac{1}{2}$	$\frac{1}{2} < x$
Sign of y''	−	+
Behavior of y	Concave down	Concave up

(a) There is a local maximum at $x = -1$.

(b) There is a local minimum at $x = 2$.

(c) There is a point of inflection at $x = \frac{1}{2}$.

19. Since $\frac{d}{dx}\left(-\frac{1}{4}x^{-4} - e^{-x}\right) = x^{-5} + e^{-x}$,

$f(x) = -\frac{1}{4}x^{-4} - e^{-x} + C.$

20. Since $\frac{d}{dx}\sec x = \sec x \tan x,\ f(x) = \sec x + C.$

21. Since $\frac{d}{dx}\left(2\ln x + \frac{1}{3}x^3 + x\right) = \frac{2}{x} + x^2 + 1,$

$f(x) = 2\ln x + \frac{1}{3}x^3 + x + C.$

22. Since $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + 2x^{1/2}\right) = \sqrt{x} + \frac{1}{\sqrt{x}},$

$f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + C.$

23. $f(x) = -\cos x + \sin x + C$

$$\begin{aligned} f(\pi) &= 3 \\ 1 + 0 + C &= 3 \\ C &= 2 \\ f(x) &= -\cos x + \sin x + 2 \end{aligned}$$

24. $f(x)=\frac{3}{4}x^{4/3}+\frac{1}{3}x^3+\frac{1}{2}x^2+x+C$

$$f(1)=0$$
$$\frac{3}{4}+\frac{1}{3}+\frac{1}{2}+1+C=0$$
$$C=-\frac{31}{12}$$
$$f(x)=\frac{3}{4}x^{4/3}+\frac{1}{3}x^3+\frac{1}{2}x^2+x-\frac{31}{12}$$

25. $v(t)=s'(t)=9.8t+5$

$$s(t)=4.9t^2+5t+C$$
$$s(0)=10$$
$$C=10$$
$$s(t)=4.9t^2+5t+10$$

26. $a(t)=v'(t)=32$

$$v(t)=32t+C_1$$
$$v(0)=20$$
$$C_1=20$$
$$v(t)=s'(t)=32t+20$$
$$s(t)=16t^2+20t+C_2$$
$$s(0)=5$$
$$C_2=5$$
$$s(t)=16t^2+20t+5$$

27. $f(x)=\tan x$

$$f'(x)=\sec^2 x$$
$$\begin{aligned}L(x)&=f\left(-\frac{\pi}{4}\right)+f'\left(-\frac{\pi}{4}\right)\left[x-\left(-\frac{\pi}{4}\right)\right]\\&=\tan\left(-\frac{\pi}{4}\right)+\sec^2\left(-\frac{\pi}{4}\right)\left(x+\frac{\pi}{4}\right)\\&=-1+2\left(x+\frac{\pi}{4}\right)\\&=2x+\frac{\pi}{2}-1\end{aligned}$$

28. $f(x)=\sec x$

$$f'(x)=\sec x\tan x$$
$$\begin{aligned}L(x)&=f\left(\frac{\pi}{4}\right)+f'\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right)\\&=\sec\left(\frac{\pi}{4}\right)+\sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right)\\&=\sqrt{2}+\sqrt{2}(1)\left(x-\frac{\pi}{4}\right)\\&=\sqrt{2}x-\frac{\pi\sqrt{2}}{4}+\sqrt{2}\end{aligned}$$

29. $f(x)=\frac{1}{1+\tan x}$

$$\begin{aligned}f'(x)&=-(1+\tan x)^{-2}(\sec^2 x)\\&=-\frac{1}{\cos^2 x(1+\tan x)^2}\end{aligned}$$
$$\begin{aligned}L(x)&=f(0)+f'(0)(x-0)\\&=1-1(x-0)\\&=-x+1\end{aligned}$$

30. $f(x)=e^x+\sin x$

$$f'(x)=e^x+\cos x$$
$$\begin{aligned}L(x)&=f(0)+f'(0)(x-0)\\&=1+2(x-0)\\&=2x+1\end{aligned}$$

31. The global minimum value of $\frac{1}{2}$ occurs at $x=2$.

32. **(a)** The values of y' and y'' are both negative where the graph is decreasing and concave down, at T.

(b) The value of y' is negative and the value of y'' is positive where the graph is decreasing and concave up, at P.

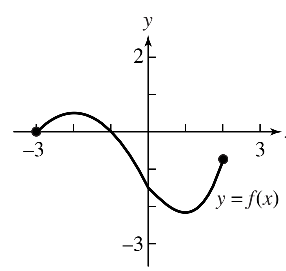
33. **(a)** The function is increasing on the interval (0, 2].

(b) The function is decreasing on the interval [−3, 0).

(c) The local extreme values occur only at the endpoints of the domain. A local maximum value of 1 occurs at $x=-3$, and a local maximum value of 3 occurs at $x=2$.

34. The 24th day

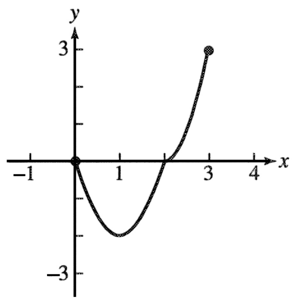
35.



36. **(a)** We know that f is decreasing on [0, 1] and increasing on [1, 3], the absolute minimum value occurs at $x = 1$ and the absolute maximum value occurs at an endpoint. Since $f(0) = 0$, $f(1) = -2$, and $f(3) = 3$, the absolute minimum value is -2 at $x = 1$ and the absolute maximum value is 3 at $x = 3$.

(b) The concavity of the graph does not change. There are no points of inflection.

(c)



37. **(a)** $f(x)$ is continuous on [0.5, 3] and differentiable on (0. 5, 3).

(b) $f'(x) = (x)\left(\frac{1}{x}\right) + (\ln x)(1) = 1 + \ln x$

Using $a = 0.5$ and $b = 3$, we solve as follows.

$$f'(c) = \frac{f(3) - f(0.5)}{3 - 0.5}$$

$$1 + \ln c = \frac{3\ln 3 - 0.5\ln 0.5}{2.5}$$

$$\ln c = \frac{\ln\left(\frac{3^3}{0.5^{0.5}}\right)}{2.5} - 1$$

$$\ln c = 0.4\ln(27\sqrt{2}) - 1$$

$$c = e^{-1}(27\sqrt{2})^{0.4}$$

$$c = e^{-1}\sqrt[5]{1458} \approx 1.579$$

(c) The slope of the line is $m = \frac{f(b) - f(a)}{b - a}$

$$= 0.4\ln\left(27\sqrt{2}\right)$$
$$= 0.2 \ln 1458,$$

and the line passes through (3, 3 ln 3). Its equation is $y = 0.2(\ln 1458)(x - 3) + 3\ln 3$, or approximately $y = 1.457x - 1.075$.

(d) The slope of the line is $m = 0.2\ln 1458$, and the line passes through

$$(c, f(c)) = (e^{-1}\sqrt[5]{1458},\ e^{-1}\sqrt[5]{1458}(-1 + 0.2\ \ln\ 1458))$$
$$\approx (1.579,\ 0.722).$$

Its equation is

$y = 0.2(\ln 1458)(x - c) + f(c)$, $y = 0.2\ln 1458(x - e^{-1}\sqrt[5]{1458}) + e^{-1}\sqrt[5]{1458}(-1 + 0.2\ln 1458)$,

$y = 0.2(\ln 1458)x - e^{-1}\sqrt[5]{1458}$, or approximately $y = 1.457x - 1.579$.

38. **(a)** $v(t) = s'(t) = 4 - 6t - 3t^2$

(b) $a(t) = v'(t) = -6 - 6t$

(c) The particle starts at position 3 moving in the positive direction, but decelerating. At approximately $t = 0.528$, it reaches position 4.128 and changes direction, beginning to move in the negative direction. After that, it continues to accelerate while moving in the negative direction.

39. **(a)** $$\begin{aligned} L(x) &= f(0) + f'(0)(x-0) \\ &= -1 + 0(x-0) \\ &= -1 \end{aligned}$$

(b) $f(0.1) \approx L(0.1) = -1$

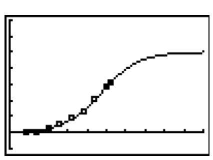
(c) Greater than the approximation in (b), since $f'(x)$ is actually positive over the interval (0, 0.1) and the estimate is based on the derivative being 0.

40. **(a)** Since $$\begin{aligned} \frac{dy}{dx} &= (x^2)(-e^{-x}) + (e^{-x})(2x) \\ &= (2x - x^2)e^{-x}, \end{aligned}$$
$dy = (2x - x^2)e^{-x}dx.$

(b) $$\begin{aligned} dy &= [2(1) - (1)^2](e^{-1})(0.01) \\ &= 0.01e^{-1} \\ &\approx 0.00368 \end{aligned}$$

41. **(a)** With some rounding, the population in thousands is given by
$y = \dfrac{502{,}191}{1 + 139.238e^{-0.02075t}}$ where $t = 0$ represents 1750.

(b)



[0, 500] by [−100,000, 700,000]

(c) $$\begin{aligned} y &= \frac{502{,}191}{1 + 139.238e^{-0.02075(300)}} \\ &\approx 393{,}709 \text{ thousand or } 393{,}709{,}000 \end{aligned}$$

(d) Using the Second Derivative, we find the maximum rate of growth about 1988. We find a point of inflection here, which shows the beginning of a decline in the rate of growth.

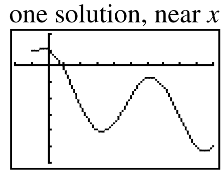
(e) The regression equation predicts a long-term maximum population of 502,191,000.

42. $f(x) = 2\cos x - \sqrt{1+x}$

$$f'(x) = -2\sin x - \frac{1}{2\sqrt{1+x}}$$

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\ &= x_n - \frac{2\cos x_n - \sqrt{1+x_n}}{-2\sin x_n - \frac{1}{2\sqrt{1+x_n}}} \end{aligned}$$

The graph of $y = f(x)$ shows that $f(x) = 0$ has one solution, near $x = 1$.



[−2, 10] by [−6, 2]

$x_1 = 1$
$x_2 \approx 0.8361848$
$x_3 \approx 0.8283814$
$x_4 \approx 0.8283608$
$x_5 \approx 0.8283608$
Solution: $x \approx 0.828361$

43. Let t represent time in seconds, where the rocket lifts off at $t = 0$. Since $a(t) = v'(t) = 20 \text{ m/sec}^2$ and $v(0) = 0$ m / sec, we have $v(t) = 20t$, and so $v(60) = 1200$ m/sec. The speed after 1 minute (60 seconds) will be 1200 m/sec.

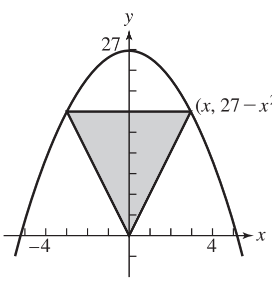
44. Let t represent time in seconds, where the rock is blasted upward at $t = 0$. Since $a(t) = v'(t) = -3.72 \text{ m/sec}^2$ and $v(0) = 93$ m/sec, we have $v(t) = -3.72t + 93$. Since $s'(t) = -3.72t + 93$ and $s(0) = 0$, we have $s(t) = -1.86t^2 + 93t$. Solving $v(t) = 0$, we find that the rock attains its maximum height at $t = 25$ sec and its height at that time is $s(25) = 1162.5$ m.

45. Note that $s = 100 - 2r$ and the sector area is given by

$$A = \pi r^2\left(\frac{s}{2\pi r}\right) = \frac{1}{2}rs = \frac{1}{2}r(100-2r) = 50r - r^2.$$

To find the domain of $A(r) = 50r - r^2$, note that $r > 0$ and $0 < s < 2\pi r$, which implies $0 < 100 - 2r < 2\pi r$, which gives $12.1 \approx \frac{50}{\pi+1} < r < 50$. Since $A'(r) = 50 - 2r$, the critical point occurs at $r = 25$. This value is in the domain and corresponds to the maximum area because $A''(r) = -2$, which is negative for all r. The greatest area is attained when $r = 25$ ft and $s = 50$ ft.

46.



For $0 < x < \sqrt{27}$, the triangle with vertices at (0, 0) and $(\pm x,\ 27 - x^2)$ has an area given by $A(x) = \frac{1}{2}(2x)(27 - x^2) = 27x - x^3$. Since $A' = 27 - 3x^2 = 3(3-x)(3+x)$ and $A'' = -6x$, the critical point in the interval $(0, \sqrt{27})$ occurs at $x = 3$ and corresponds to the maximum area because $A''(x)$ is negative in this interval. The largest possible area is $A(3) = 54$ square units.

47. If the dimensions are x ft by x ft by h ft, then the total amount of steel used is $x^2 + 4xh$ ft^2. Therefore, $x^2 + 4xh = 108$ and so $h = \frac{108 - x^2}{4x}$. The volume is given by $V(x) = x^2h = \frac{108x - x^3}{4} = 27x - 0.25x^3$. Then $V'(x) = 27 - 0.75x^2 = 0.75(6+x)(6-x)$ and $V''(x) = -1.5x$. The critical point occurs at $x = 6$, and it corresponds to the maximum volume because $V''(x) < 0$ for $x > 0$. The corresponding height is $\frac{108 - 6^2}{4(6)} = 3$ ft. The base measures 6 ft by 6 ft, and the height is 3 ft.

48. If the dimensions are x ft by x ft by h ft, then we have $x^2h = 32$ and so $h = \frac{32}{x^2}$. Neglecting the quarter-inch thickness of the steel, the area of the steel used is $A(x) = x^2 + 4xh = x^2 + \frac{128}{x}$. We can minimize the weight of the vat by minimizing this quantity. Now $A'(x) = 2x - 128x^{-2} = \frac{2}{x^2}(x^3 - 4^3)$ and $A''(x) = 2 + 256x^{-3}$. The critical point occurs at $x = 4$ and corresponds to the minimum possible area because $A''(x) > 0$ for $x > 0$. The corresponding height is $\frac{32}{4^2} = 2$ ft. The base should measure 4 ft by 4 ft, and the height should be 2 ft.

49. We have $r^2 + \left(\frac{h}{2}\right)^2 = 3$, so $r^2 = 3 - \frac{h^2}{4}$. We wish to minimize the cylinder's volume $V = \pi r^2 h = \pi\left(3 - \frac{h^2}{4}\right)h = 3\pi h - \frac{\pi h^3}{4}$ for $0 < h < 2\sqrt{3}$. Since $\frac{dV}{dh} = 3\pi - \frac{3\pi h^2}{4} = \frac{3\pi}{4}(2+h)(2-h)$ and $\frac{d^2V}{dh^2} = -\frac{3\pi h}{2}$, the critical point occurs at $h = 2$ and it corresponds to the maximum value because $\frac{d^2V}{dh^2} < 0$ for $h > 0$. The corresponding value of r is $\sqrt{3 - \frac{2^2}{4}} = \sqrt{2}$. The largest possible cylinder has height 2 and radius $\sqrt{2}$.

50. Note that, from similar cones, $\frac{r}{6} = \frac{12-h}{12}$, so $h = 12–2r$. The volume of the smaller cone is given by

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(12-2r) = 4\pi r^2 - \frac{2\pi}{3}r^3$$

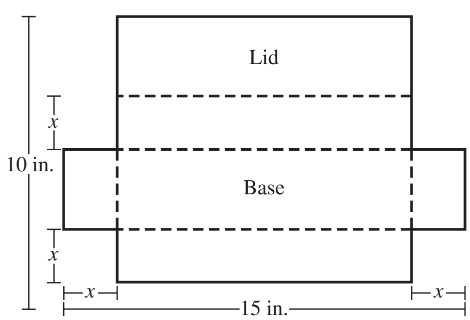
for $0 < r < 6$. Then

$\frac{dV}{dr} = 8\pi r - 2\pi r^2 = 2\pi r(4-r)$, so the critical point occurs at $r = 4$. This critical point corresponds to the maximum volume because $\frac{dV}{dr} > 0$ for

$$\begin{aligned} a - mx &= f(x) - f'(x)\cdot x \\ &= B + \frac{B}{C}\sqrt{C^2 - x^2} + \frac{Bx^2}{C\sqrt{C^2-x^2}} \\ &= B + \frac{B}{C}\sqrt{C^2 - \frac{3C^2}{4}} + \frac{B\left(\frac{3C^2}{4}\right)}{C\sqrt{C^2 - \frac{3C^2}{4}}} \\ &= B + \frac{B}{C}\left(\frac{C}{2}\right) + \frac{\frac{3BC^2}{4}}{\frac{C^2}{2}} \\ &= B + \frac{B}{2} + \frac{3B}{2} \\ &= 3B \end{aligned}$$

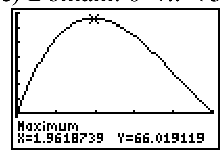
and $\frac{dV}{dr} < 0$ for $4 < r < 6$. The smaller cone has the largest possible value when $r = 4$ ft and $h = 4$ ft.

51.



(a) $V(x) = x(15-2x)(5-x)$

(b, c) Domain: $0 < x < 5$



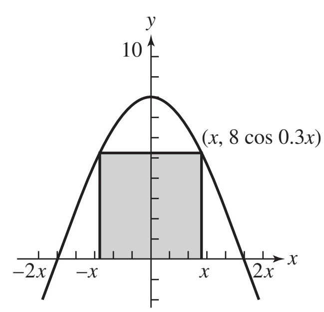
The maximum volume is approximately $66.019\ \text{in}^3$ and it occurs when $x \approx 1.962$ in.

(d) Note that $V(x) = 2x^3 - 25x^2 + 75x$, so $V'(x) = 6x^2 - 50x + 75$. Solving $V'(x) = 0$, we have

$$\begin{aligned} x &= \frac{50 \pm \sqrt{(-50)^2 - 4(6)(75)}}{2(6)} \\ &= \frac{50 \pm \sqrt{700}}{12} \\ &= \frac{50 \pm 10\sqrt{7}}{12} \\ &= \frac{25 \pm 5\sqrt{7}}{6}. \end{aligned}$$

These solutions are approximately $x \approx 1.962$ and $x = 6.371$, so the critical point in the appropriate domain occurs at $x = \frac{25 - 5\sqrt{7}}{6}$.

52.



For $0 < x < \frac{5\pi}{3}$, the area of the rectangle is given by

$A(x) = (2x)(8\cos 0.3x) = 16x\cos 0.3x.$

Then

$$\begin{aligned} A'(x) &= 16x(-0.3\sin 0.3x) + 16(\cos 0.3x)(1) \\ &= 16(\cos 0.3x - 0.3x\sin 0.3x) \end{aligned}$$

Solving $A'(x) = 0$ graphically, we find that the critical point occurs at $x \approx 2.868$ and the corresponding area is approximately 29.925 square units.

53. The cost (in thousands of dollars) is given by

$$C(x) = 40x + 30(20 - y) = 40x + 600 - 30\sqrt{x^2 - 144}.$$

Then $C'(x) = 40 - \frac{30}{2\sqrt{x^2-144}}(2x) = 40 - \frac{30x}{\sqrt{x^2-144}}.$

Solving $C'(x) = 0,$ we have:

$$\frac{30x}{\sqrt{x^2-144}} = 40$$
$$3x = 4\sqrt{x^2-144}$$
$$9x^2 = 16x^2 - 2304$$
$$2304 = 7x^2$$

Choose the positive solution:

$$x = +\frac{48}{\sqrt{7}} \approx 18.142 \text{ mi}$$

$$y = \sqrt{x^2 - 12^2} = \frac{36}{\sqrt{7}} \approx 13.607 \text{ mi}$$

54. The length of the track is given by $2x + 2\pi r$, so we have $2x + 2\pi r = 400$ and therefore $x = 200 - \pi r$. Then the area of the rectangle is

$$A(r) = 2rx = 2r(200 - \pi r) = 400r - 2\pi r^2, \text{ for } 0 < r < \frac{200}{\pi}.$$

Therefore, $A'(r) = 400 - 4\pi r$ and $A''(r) = -4\pi$, so the critical point occurs at $r = \frac{100}{\pi}$ m and this point corresponds to the maximum rectangle area because $A''(r) < 0$ for all r.

The corresponding value of x is

$$x = 200 - \pi\left(\frac{100}{\pi}\right) = 100 \text{ m}.$$

The rectangle will have the largest possible area when $x = 100$ m and $r = \frac{100}{\pi}$ m.

55. Assume the profit is k dollars per hundred grade B tires and $2k$ dollars per hundred grade A tires. Then the profit is given by

$$P(x) = 2kx + k\cdot\frac{40-10x}{5-x} = 2k\cdot\frac{(20-5x)+x(5-x)}{5-x} = 2k\cdot\frac{20-x^2}{5-x}$$

$$P'(x) = 2k\cdot\frac{(5-x)(-2x)-(20-x^2)(-1)}{(5-x)^2} = 2k\cdot\frac{x^2-10x+20}{(5-x)^2}$$

The solutions of $P'(x) = 0$ are

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)} = 5 \pm \sqrt{5},$$ so the solution in the appropriate domain is $x = 5 - \sqrt{5} \approx 2.76.$

Check the profit for the critical point and endpoints:

Critical point: $x \approx 2.76$ $P(x) \approx 11.06k$

End points: $x = 0$ $P(x) = 8k$

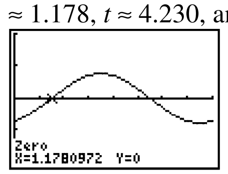
$x = 4$ $P(x) = 8k$

The highest profit is obtained when $x \approx 2.76$ and $y \approx 5.53$, which corresponds to 276 grade A tires and 553 grade B tires.

56. **(a)** The distance between the particles is $|f(t)|$ where $f(t) = -\cos t + \cos\left(t + \frac{\pi}{4}\right)$. Then

$$f'(t) = \sin t - \sin\left(t + \frac{\pi}{4}\right)$$

Solving $f'(t) = 0$ graphically, we obtain $t \approx 1.178$, $t \approx 4.230$, and so on.



$[0, 2\pi]$ by $[-2, 2]$

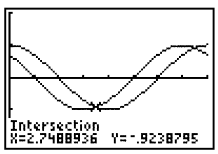
Alternatively, $f'(t) = 0$ may be solved analytically as follows.

$$f'(t) = \sin\left[\left(t+\frac{\pi}{8}\right)-\frac{\pi}{8}\right] - \sin\left[\left(t+\frac{\pi}{8}\right)+\frac{\pi}{8}\right]$$
$$= \left[\sin\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8} - \cos\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}\right] - \left[\sin\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8} + \cos\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}\right]$$
$$= -2\sin\frac{\pi}{8}\cos\left(t+\frac{\pi}{8}\right),$$

so the critical points occur when $\cos\left(t+\frac{\pi}{8}\right) = 0$, or $t = \frac{3\pi}{8} + k\pi$. At each of these values,

$f(t) = \pm 2\cos\frac{3\pi}{8} \approx \pm 0.765$ units, so the maximum distance between the particles is 0.765 unit.

(b) Solving $\cos t = \cos\left(t+\frac{\pi}{4}\right)$ graphically, we obtain $t \approx 2.749$, $t \approx 5.890$, and so on.



$[0, 2\pi]$ by $[-2, 2]$

Alternatively, this problem may be solved analytically as follows.

$$\cos t = \cos\left(t+\frac{\pi}{4}\right)$$
$$\cos\left[\left(t+\frac{\pi}{8}\right)-\frac{\pi}{8}\right] = \cos\left[\left(t+\frac{\pi}{8}\right)+\frac{\pi}{8}\right]$$
$$\cos\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8} + \sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8} = \cos\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8} - \sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}$$
$$2\sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8} = 0$$
$$\sin\left(t+\frac{\pi}{8}\right) = 0$$
$$t = \frac{7\pi}{8} + k\pi$$

The particles collide when $t = \frac{7\pi}{8} \approx 2.749$ (plus multiples of π if they keep going.)

57. The dimensions will be x in. by $10 - 2x$ in. by $16 - 2x$ in., so

$$V(x) = x(10-2x)(16-2x)$$
$$= 4x^3 - 52x^2 + 160x$$

for $0 < x < 5$.

Then $V'(x) = 12x^2 - 104x + 160 = 4(x-2)(3x-20)$, so the critical point in the correct domain is $x = 2$. This critical point corresponds to the maximum possible volume because $V'(x) > 0$ for $0 < x < 2$ and $V'(x) < 0$ for $2 < x < 5$. The box of largest volume has a height of 2 in. and a base measuring 6 in. by 12 in., and its volume is 144 in^3.

58. Step 1:
r = radius of circle
A = area of circle
Step 2:
At the instant in question, $\frac{dr}{dt} = -\frac{2}{\pi}$ m/sec and $r = 10$ m.
Step 3:
We want to find $\frac{dA}{dt}$.
Step 4:
$A = \pi r^2$
Step 5:
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$
Step 6:
$$\frac{dA}{dt} = 2\pi(10)\left(-\frac{2}{\pi}\right) = -40$$
The area is changing at the rate of $-40\ \text{m}^2/\text{sec}$.

59. Step 1:
x = x-coordinate of particle
y = y-coordinate of particle
D = distance from origin to particle
Step 2:
At the instant in question, $x = 5$ m, $y = 12$ m, $\frac{dx}{dt} = -1$ m/sec, and $\frac{dy}{dt} = -5$ m/sec.
Step 3:
We want to find $\frac{dD}{dt}$.
Step 4:
$D = \sqrt{x^2 + y^2}$
Step 5:
$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$
Step 6:
$$\frac{dD}{dt} = \frac{(5)(-1)+(12)(-5)}{\sqrt{5^2+12^2}} = -5 \text{ m/sec}$$
Since $\frac{dD}{dt}$ is negative, the particle is *approaching* the origin at the *positive* rate of 5 m/sec.

60. Step 1:
x = edge of length of cube
V = volume of cube
Step 2:
At the instant in question,
$\frac{dV}{dt} = 1200\ \text{cm}^3/\text{min}$ and $x = 20$ cm.
Step 3:
We want to find $\frac{dx}{dt}$.
Step 4:
$V = x^3$
Step 5:
$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$
Step 6:
$$1200 = 3(20)^2\frac{dx}{dt}$$
$$\frac{dx}{dt} = 1 \text{ cm/min}$$
The edge length is increasing at the rate of 1 cm/min.

61. Step 1:
x = x-coordinate of point
y = y-coordinate of point
D = distance from origin to point
Step 2:
At the instant in question, $x = 3$ and $\frac{dD}{dt} = 11$ units per sec.
Step 3:
We want to find $\frac{dx}{dt}$.
Step 4:
Since $D^2 = x^2 + y^2$ and $y = x^{3/2}$, we have $D = \sqrt{x^2 + x^3}$ for $x \ge 0$.
Step 5:
$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+x^3}}(2x+3x^2)\frac{dx}{dt} = \frac{2x+3x^2}{2x\sqrt{1+x}}\frac{dx}{dt} = \frac{3x+2}{2\sqrt{1+x}}\frac{dx}{dt}$$
Step 6:
$$11 = \frac{3(3)+2}{2\sqrt{4}}\frac{dx}{dt}$$
$$\frac{dx}{dt} = 4 \text{ units per sec}$$

62. **(a)** Since $\frac{h}{r}=\frac{10}{4}$, we may write

$h=\frac{5r}{2}$ or $r=\frac{2h}{5}$.

(b) Step 1:
h = depth of water in tank
r = radius of surface of water
V = volume of water in tank
Step 2:
At the instant in question,
$\frac{dV}{dt}=-5\text{ ft}^3/\text{min}$ and $h=6$ ft.
Step 3:
We want to find $-\frac{dh}{dt}$.
Step 4:
$V=\frac{1}{3}\pi r^2 h=\frac{4}{75}\pi h^3$
Step 5:
$\frac{dV}{dt}=\frac{4}{25}\pi h^2\frac{dh}{dt}$
Step 6:
$-5=\frac{4}{25}\pi(6)^2\frac{dh}{dt}$
$\frac{dh}{dt}=-\frac{125}{144\pi}\approx -0.276$ ft/min
Since $\frac{dh}{dt}$ is negative, the water level is *dropping* at the positive rate of ≈ 0.276 ft/min.

63. Step 1:
r= radius of outer layer of cable on the spool
θ= clockwise angle turned by spool
s= length of cable that has been unwound
Step 2:
At the instant in question, $\frac{ds}{dt}=6$ ft/sec and $r=1.2$ ft
Step 3:
We want to find $\frac{d\theta}{dt}$.
Step 4:
$s=r\theta$
Step 5:
Since r is essentially constant, $\frac{ds}{dt}=r\frac{d\theta}{dt}$
Step 6:
$6=1.2\frac{d\theta}{dt}$
$\frac{d\theta}{dt}=5$ radians/sec
The spool is turning at the rate of 5 radians per second.

64. $a(t)=v'(t)=-g=-32\text{ ft/sec}^2$
Since $v(0)=32$ ft/sec, $v(t)=s'(t)=-32t+32$.
Since $s(0)=-17$ ft, $s(t)=-16t^2+32t-17$.
The shovelful of dirt reaches its maximum height when $v(t)=0$, at $t=1$ sec. Since $s(1)=-1$, the shovelful of dirt is still below ground level at this time. There was not enough speed to get the dirt out of the hole. Duck!

65. We have $V=\frac{1}{3}\pi r^2 h$, so $\frac{dV}{dr}=\frac{2}{3}\pi rh$ and

$dV=\frac{2}{3}\pi rh\ dr$.

When the radius changes from a to $a+dr$, the volume change is approximately
$dV=\frac{2}{3}\pi ah$ dr.

66. **(a)** Let x = edge of length of cube and S = surface area of cube. Then $S=6x^2$, which means $\frac{dS}{dx}=12x$ and $dS=12x\ dx$.
We want $|dS|\le 0.02S$, which gives $|12x\ dx|\le 0.02(6x^2)$ or $|dx|\le 0.01x$. The edge should be measured with an error of no more than 1%.

(b) Let V = volume of cube. Then $V=x^3$, which means $\frac{dV}{dx}=3x^2$ and $dV=3x^2dx$.
We have $|dx|\le 0.01x$, which means $|3x^2dx|\le 3x^2(0.01x)=0.03V$, so $|dV|\le 0.03V$. The volume calculation will be accurate to within approximately 3% of the correct volume.

67. Let C = circumference, r = radius, S = surface area, and V = volume.

(a) Since $C = 2\pi r,$ we have $\dfrac{dC}{dr} = 2\pi$ and so $dC = 2\pi\, dr.$
Therefore,
$\left|\dfrac{dC}{C}\right| = \left|\dfrac{2\pi dr}{2\pi r}\right| = \left|\dfrac{dr}{r}\right| < \dfrac{0.4\,\text{cm}}{10\,\text{cm}} = 0.04$ The calculated radius will be within approximately 4% of the correct radius.

(b) Since $S = 4\pi r^2,$ we have $\dfrac{dS}{dr} = 8\pi r$ and so $dS = 8\pi r\ dr.$ Therefore,
$\left|\dfrac{dS}{S}\right| = \left|\dfrac{8\pi r\ dr}{4\pi r^2}\right| = \left|\dfrac{2\ dr}{r}\right| \le 2(0.04) = 0.08.$
The calculated surface area will be within approximately 8% of the correct surface area.

(c) Since $V = \dfrac{4}{3}\pi r^3,$ we have $\dfrac{dV}{dr} = 4\pi r^2$ and so $dV = 4\pi r^2\ dr.$ Therefore

$$\left|\frac{dV}{V}\right| = \left|\frac{4\pi r^2\ dr}{\frac{4}{3}\pi r^3}\right| = \left|\frac{3\ dr}{r}\right| \le 3(0.04) = 0.12.$$

The calculated volume will be within approximately 12% of the correct volume.

68. By similar triangles, we have $\dfrac{a}{6} = \dfrac{a+20}{h},$ which gives $ah = 6a + 120$ or $h = 6 + 120a^{-1}$
The height of the lamp post is approximately $6 + 120(15)^{-1} = 14$ ft. The estimated error in measuring a was $|da| \le 1$ in. $= \dfrac{1}{12}$ ft. Since $\dfrac{dh}{da} = -120a^{-2},$ we have
$|dh| = \left|-120a^{-2}\ da\right| \le 120(15)^{-2}\left(\dfrac{1}{12}\right) = \dfrac{2}{45}$ ft,
so the estimated possible error is
$\pm\dfrac{2}{45}$ ft or $\pm\dfrac{8}{15}$ in.

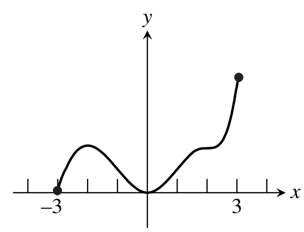
69. $\dfrac{dy}{dx} = 2\sin x\cos x - 3.$ Since $\sin x$ and $\cos x$ are both between 1and -1, the value of $2\sin x\cos x$ is never greater than 2. Therefore,
$\dfrac{dy}{dx} \le 2 - 3 = -1$ for all values of x.
Since $\dfrac{dy}{dx}$ is always negative, the function decreases on every interval.

70. **(a)** f has a relative maximum at $x = -2$. This is where $f'(x) = 0,$ causing f' to go from positive to negative.

(b) f has a relative minimum at $x = 0$. This is where $f'(x) = 0,$ causing f' to go from negative to positive.

(c) The graph of f is concave up on $(-1, 1)$ and on $(2, 3)$. These are the intervals on which the derivative of f is increasing.

(d)



71. **(a)**

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r dr$$
$$\frac{dA}{dt} = 2\pi(2)\left(\frac{1}{3}\right) = \frac{4}{3}\pi\frac{\text{in.}^2}{\text{sec}}$$

(b) $V = \dfrac{1}{3}\pi\,(2^2)h = 8\pi \Rightarrow h = 6$

$$4\pi = \frac{dV}{dt} = \frac{1}{3}\pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$
$$4 = \frac{1}{3}\left(2(2)6\left(\frac{1}{3}\right) + 2^2\frac{dh}{dt}\right)$$
$$12 = \left(8 + 4\frac{dh}{dt}\right)$$
$$1 = \frac{dh}{dt}$$

(c) $\dfrac{dA}{dh} = \dfrac{\frac{4}{3}\pi}{1} = \dfrac{4}{3}\pi\dfrac{\text{in.}^2}{\text{in.}}$

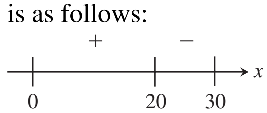
72. **(a)** $V = \pi\left(\frac{a}{2\pi}\right)^2 b,$ and $b = \frac{60-2a}{4} = 15 - \frac{a}{2},$

so $V = \frac{30a^2 - a^3}{8\pi}.$

Thus

$\frac{dV}{da} = \frac{1}{8\pi}\left(60a - 3a^2\right) = \frac{3}{8\pi}a(20-a).$ The relevant domain for a in this problem is (0, 30), so $a = 20$ is the only critical number. The cylinder of maximum volume is formed when $a = 20$ and $b = 5$.

(b) The sign graph for the derivative $\frac{dV}{da} = \frac{3}{8\pi}a(20-a)$ on the interval (0, 30) is as follows:



By the First Derivative Test, there is a maximum at $a = 20$.